ChE515: Mathematical Methods in Chemical Engineering
Problem Set 3: Stability Analysis
Due Date: October 20, 2003.

1. **VM 2.11**(a)., (d)., (e). Verify that stability characteristics of the fixed point ($\boldsymbol{x} = 0$) is consistent with the classification using *trace*(***A***) – *det*(***A***) given in Figure 2.21.

2. **VM 2.13**. Determine stability of the fixed point using the eigenvalues of the Jacobian matrix.

3. **VM 2.14**. In order to simplify the algebra, you may set the absorption column parameters *L*, *G* and *h* to unity and $\alpha = 2$ and $x_f + y_f = 1$.

4. **VM 2.17**. Note that this CSTR operates under isothermal conditions.

5. **VM 2.26. Hopf bifurcation**.

6. **VM 2.32. Brusselator**. Prigogine has written a number of popular science level articles/books on nonlinear dynamics, pattern formation and chaos that make interesting reading.

7. **Turing model/Chemical Basis of Morphogenesis**: Consider the nonlinear reaction-diffusion equations for concentrations *u* and *v* of two chemical species in a spatial domain *D*

$$\frac{\partial u}{\partial t} = f(u,v) + \nabla^2 u$$
$$\frac{\partial v}{\partial t} = g(u,v) + d\nabla^2 v$$

with Neumann (no-flux) boundary conditions and $d > 1$. Let (U, V) denote the *uniform* steady state such that $f(U,V) = g(U,V) = 0$. Let

$$u'(\boldsymbol{x},t) = \hat{u}\exp(st + i\boldsymbol{k}.\boldsymbol{x}) \text{ and } v'(\boldsymbol{x},t) = \hat{v}\exp(st + i\boldsymbol{k}.\boldsymbol{x})$$

denote infinitesimally small perturbations of the steady states of *u* and *v*, respectively with *s* denoting the eigenvalues associated with a real wavenumber $k = |\boldsymbol{k}|$. Show that the eigenvalues of the linearized problem are given by

$$\det\begin{bmatrix} s + k^2 - f_u(U,V) & -f_v(U,V) \\ -g_u(U,V) & s + dk^2 - g_v(U,V) \end{bmatrix} = 0,$$

where subscript to a function denotes its partial derivative.

8. **Swift-Hohenberg** model for nonlinear Rayleigh-Bénard convection (explained in chapter 2 of handout on pattern formation), proposed the following partial differential equation for the state *u* of the system.

$$\frac{\partial u}{\partial t} = (R - (\frac{\partial}{\partial x^2} + 1)^2)u - u^3 \text{ for } -\infty < x < \infty,\ t \geq 0,$$

where R is a positive parameter (Rayleigh number). The rest state is represented by the null solution $u = 0$. Linearize the above PDE and using the method of normal modes, i.e., letting the perturbation $u' \propto \exp(st + ikx)$ for real wavenumbers k, find s as a function of R and k. Plot the values of R vs. k for which the system is marginally stable. This curve is called the *neutral stability diagram*. Explain its physical significance.

9. **Reading assignment: Section 2.23.2 VM on Lorenz equations**. Problem 2.37 also provides a nice example of a system that leads to chaotic dynamics.