ChE515: "Warm Up" Problems

1. Solve the following systems of equations by using Gaussian elimination.

(a). $x_1+x_2+x_3 = 3$; $2x_1+x_3 = 3$; $x_1+x_2-x_3 = 1$

(b). $x_1+x_2+x_3 = 3$; $2x_1+x_3 = 3$; $4x_1+2x_3 = 6$

(c). $x_1+x_2+x_3 = 3$; $2x_1+x_3 = 3$; $4x_1+2x_3 = 3$

2. Find the eigenvalues and the corresponding eigenvectors of the following matrices.

(a). $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

(b). $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3. Solve the following ordinary differential equations (ODEs). Indicate the order of the ODE and whether the ODE is linear (or nonlinear), homogeneous (or non-homogeneous). Whenever possible, describe a physical process that can be modeled by the ODE.

(a). $\frac{dy}{dx} - \frac{2}{x}y = x$

(b). $(2xy^2 + 2)dx + (2x^2y + 4y)dy = 0$

(c). $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$

(d). $\frac{dy}{dx} + P(x)y = Q(x)y^n, \quad n \neq 1$

(e). $\frac{dy}{dx} = P(x)y^2 + Q(x)y + R(x), \; P(x) = -1$

(f). $D\frac{d^2C}{dx^2} - V\frac{dC}{dx} - kC = 0$

(g). $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$

(h). $\frac{d^2y}{dx^2} - y = x^2$

(i). $4x\frac{d^2y}{dx^2}+6\frac{dy}{dx}+y=f(x)$ (Review: Sections 3.1-3.5 in V&M)

4. Solve the following Initial Value Problem (IVP) by using the method of Laplace transforms (Review sections 8.4.1 and 8.4.2 in V&M). Describe a physical process modeled by the IVP.

$$\tau\frac{dy}{dt}+y=f(t)$$
$$y(0)=A$$

5. Solve the following Boundary Value Problems (BVPs). Describe the physical process modeled.

(a).

$$r\frac{d^2C}{dr^2}+\frac{dC}{dr}-\alpha^2rC=0,\quad 0<r<1$$
$$C(r=1)=C_0$$
$$\frac{dC}{dr}(r=0)=0$$

(b).

$$\frac{d^2C}{dx^2}-\phi^2C=0,\ 0<x<1$$
$$C(x=1)=1$$
$$\frac{dC}{dx}(x=0)=0$$

6. Solve the following differential Eigenvalue Problem (EVP).

$$\frac{d^2C}{dx^2}+\lambda C=0,\ 0<x<1$$
$$C(x=1)=0$$
$$\frac{dC}{dx}(x=0)=0$$

7. Solve, by using the method of separation of variables, the following initial boundary value problem (IBVP). Describe the physical process modeled.

$$\frac{\partial y}{\partial t}=\frac{\partial^2y}{\partial x^2},\quad t>0,\ 0<x<1$$
$$y(t=0,x)=f(x)$$
$$y(t,x=0)=1$$
$$y(t,x=1)=0$$